

# 8.7

## Volume of a Sphere

The annual Gatineau Hot Air Balloon Festival has been held since 1988 in Gatineau, Québec. Hot air balloons come in a variety of shapes. One special shape that appeared at the festival was a soccer ball. This soccer ball was about 17.4 m in diameter, and could hold 385 696 ordinary soccer balls inside it.

In this section, you will develop a formula for the volume of any sphere.



### Investigate



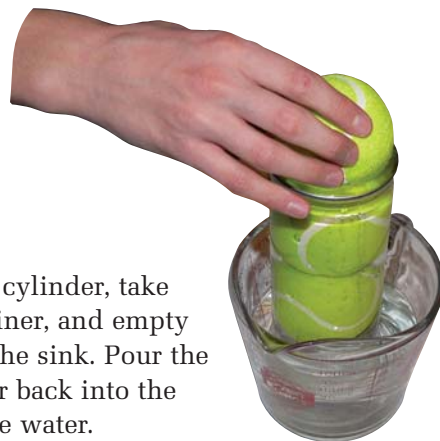
#### Tools

- a cylindrical container that just holds three tennis balls
- three old tennis balls
- water
- a container to catch the overflow water

#### How can you model the volume of a sphere?

Work in small groups.

1. Before you take any measurements, estimate the volume of one tennis ball.
2. a) Measure the diameter of the cylinder. It should be almost the same as the diameter of the tennis ball.  
b) Measure the height of the cylinder. It should be almost the same as three times the diameter of the tennis ball.
3. Place the cylinder in the overflow container and fill the cylinder with water.
4. Slowly place the three tennis balls inside the cylinder, one at a time, allowing the water to overflow into the container. Push the balls down to the bottom of the cylinder.
5. Remove the tennis balls from the cylinder, take the can out of the overflow container, and empty the water from the cylinder into the sink. Pour the water from the overflow container back into the cylinder. Measure the depth of the water.



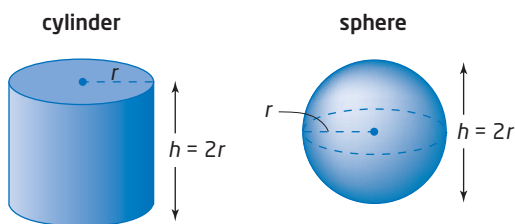
6. What fraction of the cylinder is filled with water? How does the volume of this displaced water compare to the volume of the three tennis balls?
7. If the cylinder were only big enough to hold one tennis ball, what fraction of the can would be filled with water?
8. **Reflect** How does the volume of a sphere compare to the volume of a cylinder? How would you calculate the volume of one tennis ball?
9. Use your method to calculate the volume of one tennis ball. How does your answer compare to your estimate?

### Example 1 Volume of Pluto

Pluto is the smallest planet in the solar system. The diameter of Pluto is approximately 2290 km. Calculate the volume of Pluto.

#### Solution

The volume of a sphere is two thirds the volume of a cylinder with the same radius and a height equal to the diameter of the sphere. If the sphere has radius  $r$ , then the cylinder has a base radius  $r$  and height  $2r$ .



$$\begin{aligned}
 V_{\text{sphere}} &= \frac{2}{3} (\text{volume of a cylinder}) \\
 &= \frac{2}{3} \pi r^2 h \\
 &= \frac{2}{3} \pi (r^2)(2r) \\
 &= \frac{4}{3} \pi r^3
 \end{aligned}$$

The radius is one half the diameter. The radius of Pluto is 1145 km. Use the formula for the volume of a sphere.

$$\begin{aligned}
 V_{\text{sphere}} &= \frac{4}{3} \pi r^3 \\
 &= \frac{4}{3} \pi (1145)^3 \\
 &\doteq 6\,300\,000\,000
 \end{aligned}$$

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4/3*pi*1145^3
6287891937
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The volume of Pluto is approximately 6 300 000 000 km<sup>3</sup>.

#### Did You Know?

Jupiter is the largest planet, with an equatorial diameter of 143 884 km. Its diameter is more than 11 times that of Earth.

### Did You Know?

Since medieval times quartz crystal balls have been used in attempts to divine the future. Gemstone spheres are thought to have healing powers, with each gemstone having a different therapeutic energy.

## Example 2 Package a Gemstone

A spherical gemstone just fits inside a plastic cube with edges 10 cm.

- Calculate the volume of the gemstone, to the nearest cubic centimetre.
- How much empty space is there?

### Solution

- The diameter of the gemstone is about 10 cm, so its radius is 5 cm.

$$\begin{aligned}V_{\text{sphere}} &= \frac{4}{3} \pi r^3 \\&= \frac{4}{3} \pi (5)^3 \quad \text{Estimate: } \frac{4}{3}(3)5^3 = 4(125) \\&\doteq 524 \qquad \qquad \qquad = 500\end{aligned}$$

The volume of the gemstone is about 524 cm<sup>3</sup>.

- Determine the volume of the cube.

$$\begin{aligned}V_{\text{cube}} &= s^3 \\&= 10^3 \\&= 1000\end{aligned}$$

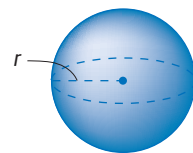
The empty space is the difference in the volumes of the cube and the gemstone.

$$\begin{aligned}V_{\text{empty space}} &= V_{\text{cube}} - V_{\text{sphere}} \\&= 1000 - 524 \\&= 476\end{aligned}$$

There is about 476 cm<sup>3</sup> of empty space in the box.

### Key Concepts

- The volume of a sphere with radius  $r$  is given by the formula  $V_{\text{sphere}} = \frac{4}{3} \pi r^3$ .
- You can calculate the empty space in a container by subtracting the volume of the object from the volume of the container in which it is packaged.



## Communicate Your Understanding

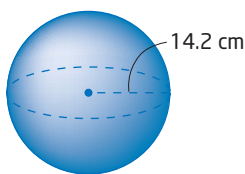
- C1** Describe how you would determine the volume of a sphere if you knew its surface area.
- C2** How is the volume of a sphere affected if you double the radius?

## Practise

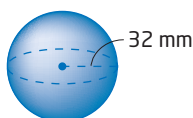
For help with questions 1 to 3, see Example 1.

1. Calculate the volume of each sphere. Round to the nearest cubic unit.

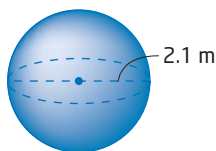
a)



b)



c)



2. A golf ball has a diameter of 4.3 cm. Calculate its volume, to the nearest cubic centimetre.



3. Hailstones thought to be the size of baseballs killed hundreds of people and cattle in the Moradabad and Beheri districts of India in 1888. The hailstones had a reported diameter of 8 cm. What was the volume of each one, to the nearest cubic centimetre?



For help with question 4, see Example 2.

4. A table tennis ball just fits inside a plastic cube with edges 40 mm.
- Calculate the volume of the table tennis ball, to the nearest cubic millimetre.
  - Calculate the volume of the cube.
  - Determine the amount of empty space.

## Connect and Apply

5. The largest lollipop ever made had a diameter of 140.3 cm and was made for a festival in Gränna, Sweden, on July 27, 2003.
- If a spherical lollipop with diameter 4 cm has a mass of 50 g, what was the mass of this giant lollipop to the nearest kilogram?
  - Describe any assumptions you have made.
6. **Chapter Problem** Emily orders a spherical gazing ball for one of her customers. It is packaged tightly in a cylindrical container with a base radius of 30 cm and a height of 60 cm.
- Calculate the volume of the sphere, to the nearest cubic centimetre.
  - Calculate the volume of the cylindrical container, to the nearest cubic centimetre.
  - What is the ratio of the volume of the sphere to the volume of the container?
  - Is this ratio consistent for any sphere that just fits inside a cylinder? Explain your reasoning.
7. Golf balls are stacked three high in a rectangular prism package. The diameter of one ball is 4.3 cm. What is the minimum amount of material needed to make the box?



8. A cylindrical silo has a hemispherical top (half of a sphere). The cylinder has a height of 20 m and a base diameter of 6.5 m.
- Estimate the total volume of the silo.
  - Calculate the total volume, to the nearest cubic metre.
  - The silo should be filled to no more than 80% capacity to allow for air circulation. How much grain can be put in the silo?
  - A truck with a bin measuring 7 m by 3 m by 2.5 m delivers grain to the farm. How many truckloads would fill the silo to its recommended capacity?
9. The tank of a propane tank truck is in the shape of a cylinder with a hemisphere at both ends. The tank has a radius of 2 m and a total length of 10.2 m. Calculate the volume of the tank, to the nearest cubic metre.



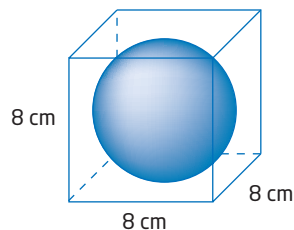
10. Estimate how many basketballs would fit into your classroom. Explain your reasoning and estimation techniques and describe any assumptions you have made. Compare your answer with that of a classmate. Are your answers close? If not, whose answer is a more reasonable estimate and why?

### Achievement Check

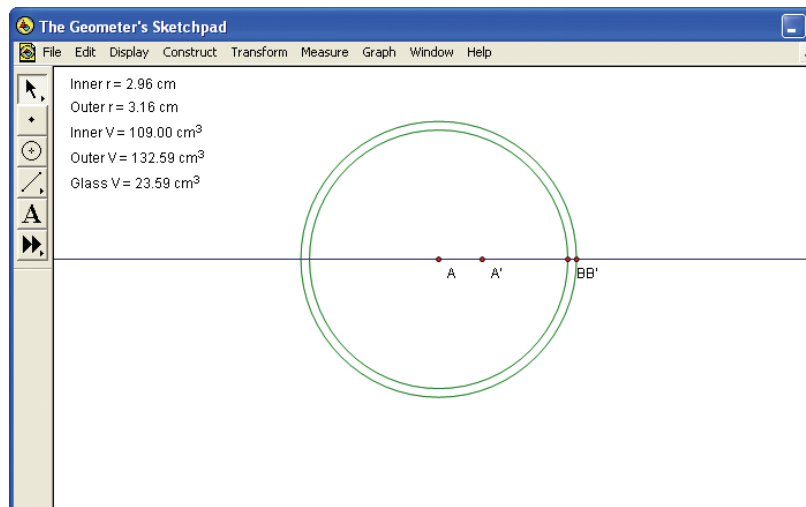
11. The T-Ball company is considering packaging two tennis balls that are 8.5 cm in diameter in a cylinder or in a square-based prism.
- What are the dimensions and volumes of the two containers?
  - How much empty space would there be in each container?
  - What factors should the T-Ball company consider in choosing the package design? Justify your choices.

### Extend

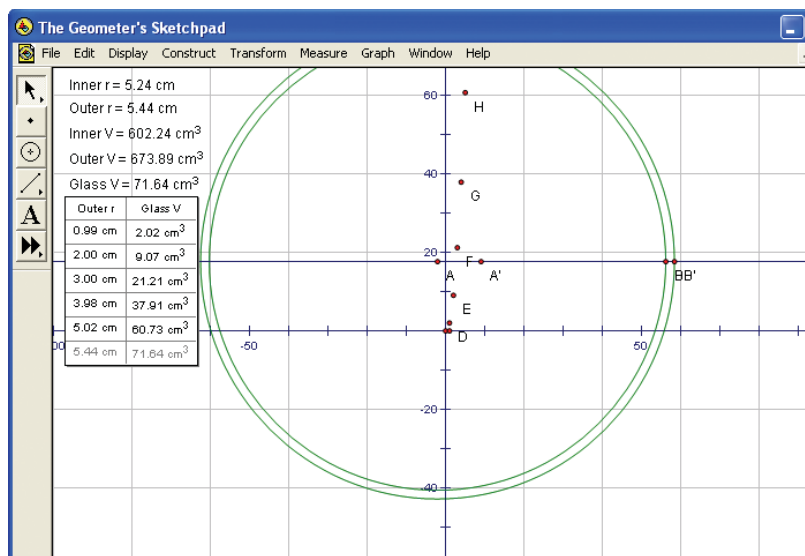
12. Estimate and then calculate the radius of a sphere with a volume of  $600 \text{ cm}^3$ .
13. **Use Technology** Graph  $V = \frac{4}{3} \pi r^3$  using a graphing calculator.
- Use the TRACE feature to determine the volume of a sphere with a radius of 6.2 cm.
  - Check your answer to question 12 by using the TRACE feature to approximate the radius of a sphere with a volume of  $600 \text{ cm}^3$ .
14. If the surface area of a sphere is doubled from  $250 \text{ cm}^2$  to  $500 \text{ cm}^2$ , by what factor does its volume increase?
15. A sphere just fits inside a cube with sides of length 8 cm.
- Estimate the ratio of the volume of the sphere to the volume of the cube.
  - Calculate the volumes of the sphere and the cube and their ratio.
  - How does your answer compare to your estimate?
16. Which has the larger volume: a sphere of radius  $r$  or a cube with edges of length  $2r$ ?



- 17. Use Technology** Use *The Geometer's Sketchpad*® to investigate how the volume of glass required to make a spherical light bulb of constant thickness 0.2 cm changes as the radius of the light bulb changes. Since *The Geometer's Sketchpad*® cannot easily show three-dimensional objects, represent the spherical light bulb with two concentric circles 0.2 cm apart.
- From the **Edit** menu, choose **Preferences**. Click on the **Text** tab. Ensure that **For All New Points** is checked.
  - Draw a point A. Select point A. From the **Transform** menu, choose **Translate**. Ensure that the **Polar**, **Fixed Distance**, and **Fixed Angle** radio buttons are on. Change the distance to 1 cm and the angle to  $0^\circ$ . Click on **Translate**. Point A' will appear 1 cm to the right of point A. Construct a line through points A and A'.
  - Draw a circle with centre A and radius AB such that point B is to the right of A', and on the line.
  - With point B selected, choose **Translate** from the **Transform** menu. Ensure that the **Polar**, **Fixed Distance**, and **Fixed Angle** radio buttons are on. Change the distance to 0.2 cm and the angle to  $0^\circ$ . Click on **Translate**. Point B' will appear 0.2 cm to the right of point B.
  - Draw a circle with centre A and radius AB'.
  - Measure the inner radius AB. Select this measurement. Right click and choose **Label Distance Measurement** from the drop-down menu. Change the label to **Inner r**.
  - Measure the outer radius AB'. Change the label to **Outer r**.
  - Select **Inner r**. From the **Measure** menu, choose **Calculate**. Enter the formula  $4 \div 3 * \pi * \text{Inner r}^3$  by selecting **Inner r** from the **Values** drop-down menu on the calculator. Change the label to **Inner V**. This is the volume of the sphere inside the light bulb.
  - Select **Outer r**. From the **Measure** menu, choose **Calculate**. Enter the formula  $4 \div 3 * \pi * \text{Outer r}^3$  by selecting **Outer r** from the **Values** drop-down menu on the calculator. Change the label to **Outer V**. This is the outer volume of the light bulb.



- Calculate the value **Outer V** – **Inner V**. Change the label to **Glass V**. This is the volume of glass required to make the light bulb.
- Select **Outer r** and then **Glass V**. From the **Graph** menu, choose **Tabulate**. Move the table to a convenient location. Move point B, and note how the values in the table change.
- Adjust the value of **Outer r** to about 1 cm. Select the table. From the **Graph** menu, choose **Add Table Data**. Click on **OK**. Adjust **Outer r** to about 2 cm. Choose **Add Table Data** again. Continue until you have five sets of data.
- From the **Graph** menu, choose **Plot Table Data**. You will see a graph of the data that you have collected.



Describe the relationship in mathematical terms.

- 18. Math Contest** A cylinder has radius 6 cm and height 6 cm. A cone has radius 6 cm and height 6 cm. A sphere has radius 6 cm. Order the figures from least volume to greatest volume. Select the correct order.

- A** cone, sphere, cylinder
- B** cone, cylinder, sphere
- C** sphere, cone, cylinder
- D** cylinder, sphere, cone
- E** cylinder, cone, sphere

- 19. Math Contest** A dozen of Terry's favourite golf balls are sold in a rectangular box. Each ball has a diameter of 4 cm. Determine the volume of empty space in the box of golf balls.

